Just in Time 17

EQUATION-SOLVING PRINCIPLES

For any real numbers a, b, and c:

***The Addition Principle*:** If $a = b$ is true, then $a + c = b + c$ is true.

***The Multiplication Principle*:** If $a = b$ is true, then $ac = bc$ is true.

Note to the student and the instructor: We assume that students come to a College Algebra course with some equation-solving skills from their study of Intermediate Algebra. Thus a portion of the material in this section might be considered by some to be review in nature. We present this material here in order to use linear functions, with which students are familiar, to lay the groundwork for zeros of higher-order polynomial functions and their connection to solutions of equations and x-intercepts of graphs.

EXAMPLE 1 Solve: $\frac{3}{4}x - 1 = \frac{7}{5}$.

Solution When we have an equation that contains fractions, it is often convenient to multiply on both sides of the equation by the least common denominator (LCD) of the fractions in order to clear the equation of fractions. We have

$$\frac{3}{4}x - 1 = \frac{7}{5}$$ The LCD is $4 \cdot 5$, or 20.

$$20\left(\frac{3}{4}x - 1\right) = 20 \cdot \frac{7}{5}$$ Multiplying by the LCD on both sides to clear fractions

$$20 \cdot \frac{3}{4}x - 20 \cdot 1 = 28$$

$$15x - 20 = 28$$

$$15x - 20 + 20 = 28 + 20$$ Using the addition principle to add 20 on both sides

$$15x = 48$$

$$\frac{15x}{15} = \frac{48}{15}$$ Using the multiplication principle to multiply by $\frac{1}{15}$, or divide by 15, on both sides

$$x = \frac{48}{15}$$

$$x = \frac{16}{5}.$$ Simplifying. Note that $\frac{3}{4}x - 1 = \frac{7}{5}$ and $x = \frac{16}{5}$ are equivalent equations.

Check:

$$\frac{3}{4}x - 1 = \frac{7}{5}$$

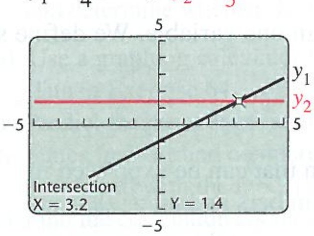
$\frac{3}{4} \cdot \frac{16}{5} - 1$?	$\frac{7}{5}$
$\frac{12}{5} - \frac{5}{5}$	
$\frac{7}{5}$	$\frac{7}{5}$ TRUE

Substituting $\frac{16}{5}$ for x

The solution is $\frac{16}{5}$.

Now Try Exercise 15.

$y_1 = \frac{3}{4}x - 1$, $y_2 = \frac{7}{5}$



We can use the INTERSECT feature on a graphing calculator to solve equations. We call this the **Intersect method**. To use the Intersect method to solve the equation in Example 1, for instance, we graph $y_1 = \frac{3}{4}x - 1$ and $y_2 = \frac{7}{5}$. The value of x for which $y_1 = y_2$ is the solution of the equation $\frac{3}{4}x - 1 = \frac{7}{5}$. This value of x is the first coordinate of the point of intersection of the graphs of y_1 and y_2. Using the INTERSECT feature, we find that the first coordinate of this point is 3.2. We can find fraction notation for the solution by using the ▶FRAC feature. The solution is 3.2, or $\frac{16}{5}$.

EXAMPLE 2 Solve: $2(5 - 3x) = 8 - 3(x + 2)$.

Algebraic Solution

We have

$$2(5 - 3x) = 8 - 3(x + 2)$$

$$10 - 6x = 8 - 3x - 6 \quad \text{Using the distributive property}$$

$$10 - 6x = 2 - 3x \quad \text{Collecting like terms}$$

$$10 - 6x + 6x = 2 - 3x + 6x \quad \text{Using the addition principle to add } 6x \text{ on both sides}$$

$$10 = 2 + 3x$$

$$10 - 2 = 2 + 3x - 2 \quad \text{Using the addition principle to add } -2\text{, or subtract 2, on both sides}$$

$$8 = 3x$$

$$\frac{8}{3} = \frac{3x}{3} \quad \text{Using the multiplication principle to multiply by } \tfrac{1}{3}\text{, or divide by 3, on both sides}$$

$$\frac{8}{3} = x.$$

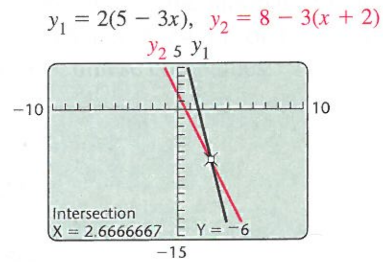
Check:

$2(5 - 3x) = 8 - 3(x + 2)$		
$2\left(5 - 3 \cdot \frac{8}{3}\right)$	? $8 - 3\left(\frac{8}{3} + 2\right)$	**Substituting $\frac{8}{3}$ for x**
$2(5 - 8)$	$8 - 3\left(\frac{14}{3}\right)$	
$2(-3)$	$8 - 14$	
-6	-6	TRUE

The solution is $\frac{8}{3}$.

Graphical Solution

We graph $y_1 = 2(5 - 3x)$ and $y_2 = 8 - 3(x + 2)$. The first coordinate of the point of intersection of the graphs is the value of x for which $2(5 - 3x) = 8 - 3(x + 2)$ and is thus the solution of the equation.



The solution is approximately 2.6666667.

We can find fraction notation for the exact solution by using the ▶FRAC feature. The solution is $\frac{8}{3}$.

X▶Frac
8/3

Now Try Exercise 27.

X	Y1	Y2
2.6667	−6	−6
X =		

We can use the TABLE feature on a graphing calculator, set in ASK mode, to check the solutions of equations. In Example 2, for instance, we let $y_1 = 2(5 - 3x)$ and $y_2 = 8 - 3(x + 2)$. When $\frac{8}{3}$ is entered for x, we see that $y_1 = y_2$, or $2(5 - 3x) = 8 - 3(x + 2)$. Thus, $\frac{8}{3}$ is the solution of the equation. (Note that the calculator converts $\frac{8}{3}$ to decimal notation in the table.)

Special Cases

Some equations have *no* solution.

EXAMPLE 3 Solve: $-24x + 7 = 17 - 24x$.

Solution We have

$$-24x + 7 = 17 - 24x$$

$$24x - 24x + 7 = 24x + 17 - 24x \quad \text{Adding } 24x$$

$$7 = 17. \quad \text{We get a false equation.}$$

No matter what number we substitute for x, we get a false equation. Thus the equation has *no* solution.

Now Try Exercise 11.